# Equilibration Time Scales in Homogeneous Bose-Einstein Condensate Dynamics

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We study the nonequilibrium growth of a weakly interacting homogeneous Bose gas after a quench from a high-temperature state to a temperature below the Bose-Einstein critical condensation temperature. We quantitatively characterize the departure from thermal equilibrium and observe the presence of *two equilibration time scales*. The equilibration times are shown to be inversely proportional to the density. PACS number(s): 05.70.Ln, 05.70.Fh, 64.60.Ak

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The nonequilibrium evolution of a Bose-Einstein condensate is a fascinating subject not only on its own but also because it allows for experimentally probing general ideas concerning the dynamics of phase transitions. Since the first experiments with dilute atomic gases led to the accomplishment of Bose-Einstein condensation [1], a great theoretical and experimental effort has been put in the investigation of different aspects of equilibrium and dynamical properties of the Bose-Einstein condensate [2]. To date, however, a comprehensive description and complete understanding of the nonequilibrium behavior of the condensate is still lacking. In this context, only a few experiments involving the study of the *growth* of the condensate in trapped dilute atomic gases [3,4] have been performed. From a theoretical standpoint, different growth equations, based on the Boltzmann equation applied to inhomogeneous trapped Bose gases, were proposed [5,6].

Recently, some of us proposed a method based on the closed-time path nonequilibrium field theoretical technique to study the growth of the *homogeneous* Bose condensate [7]. We pointed out the fundamental role played by the non-condensate fluctuations in the dynamics of the condensation growth, and were able to describe the condensate as well as the non-condensate components of the gas in a unified microscopic formalism without introducing any phenomenological parameter. However, from a conceptual point of view, the growth of the condensate is not the only observable that characterizes the out-of-

equilibrium dynamics of a bosonic system. In fact, to fully describe the out-of-equilibrium dynamics it would be necessary to compute higher-order correlation functions at two different times.

Hence, an important issue that deserves a more detailed study concerns the nonequilibrium process following a rapid quench of the vapor of dilute atomic gases, from a state well above the condensation temperature to a temperature below critical. Experimentally, this can be accomplished by the combined effect of a magnetic trap followed by a rapid RF sweep, as was done in the MIT experiment described in Ref. [3,4]. The final equilibrium state of the system depends strongly on the nonequilibrium evolution after the quench process. Therefore, it is imperative to have a formalism that is able to provide not only a description of the formation of the condensate but also to quantify the nonequilibrium behavior of the whole system. In another context, Gagne and Gleiser [8] have proposed a simple way to reliably measure the departure from thermal equilibrium of a symmetry-broken system which undergoes a temperature quench. The method makes use of the absolute value of the rate of change of the momentum-integrated structure function, $\Delta S_{\rm tot}(t)$, which we will properly define for our case (see Eq. (7) below). The main idea is that for $t < 0$ (*i.e.,* before the quench) the system is in thermal equilibrium and $\Delta S_{\rm tot}(t) \approx 0$, since in this regime each single field mode does not change in time significantly. Similarly, for large times the system returns to equilibrium (with a different temperature) and this quantity again approaches zero. Therefore, any departure from zero of $\Delta S_{\rm tot}(t)$ is a measure of the out of equilibrium regime. In other words, $\Delta S_{\rm tot}(t)$ behaves as an "order parameter" that is sensitive to the out-of-equilibrium evolution of the system. In Ref. [8] this quantity was evaluated for a simple quenched relativistic model and exhibited a peaked structure. It was shown that the height of the peak measures the departure of the system from thermal equilibrium whereas the location of the peak is a precise measure of the equilibration time scale, $\tau_{eq}$.

In this work, we apply the formalism developed in Ref. [7] to evaluate $\Delta S_{\rm tot}(t)$ in order to study the nonequilibrium behavior following a quench in a weakly-interacting homogeneous dilute Bose gas. The system is assumed to be taken from a high temperature equilibrium state, well above the critical temperature of Bose-Einstein condensation, $T_i \gg T_c$, to an unstable state in a thermal bath at a temperature $T < T_c$. Although this may be a strong

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idealization of the real setup of recent experiments involving the evolution of dilute atomic gases subject to a confining potential, we nevertheless expect that this analysis will provide important qualitative insights to the actual nonuniform systems. In fact, this approach should generically be applicable to the dynamics of trapped atomic gases in the central region of wide traps. As we will show below, our results reproduce all qualitative aspects of the condensate growth.

Our main result is the determination of a temperature range in which there are clearly *two distinct equilibration time scales*. One of the equilibration times is essentially determined by thermal fluctuations dominated by pair correlations, whereas the other is governed by the condensate growth. We also obtain numerically a scaling behavior of these quantities with the density.

Following Ref. [7] we consider a field theory effective model for a nonrelativistic complex Bose field, $\phi(\mathbf{x}, t)$, with mass $m$, and a hard core interaction potential, $V(\mathbf{x} - \mathbf{x}') = g\delta(\mathbf{x} - \mathbf{x}')$, where the coupling constant, $g$, is related to the $s$-wave scattering length, $a$, as $g = 4\pi\hbar^2 a/m$. We use the standard decomposition [9] of the fields $(\phi, \phi^*)$ into a condensate (uniform) part $(\varphi_0, \varphi_0^*)$ and a fluctuation (nonuniform) part $(\varphi, \varphi^*)$ that describes the atoms (excitations) outside the condensate, as $\phi(\mathbf{x}, t) = \varphi_0(t) + \varphi(\mathbf{x}, t)$ and $\phi^*(\mathbf{x}, t) = \varphi_0^*(t) + \varphi^*(\mathbf{x}, t)$, where we have assumed a homogeneous condensate. Particle conservation enforces the constraint on the total density,

$$n = |\varphi_0(t)|^2 + \langle \varphi^* \varphi \rangle , \qquad (1)$$

carried throughout the dynamics. Treating the interaction terms in the self-consistent Hartree-Fock (HF), which is also called Hartree-Fock-Bogoliubov approximation, we can deduce the following equation of motion for the field fluctuations:

$$i\frac{\partial \varphi}{\partial t} = \left( -\frac{\nabla^2}{2m} - \mu \right) \varphi + 2gn\varphi + g(\varphi_0^2(t) + \langle \varphi\varphi \rangle)\varphi^*, \quad (2)$$

and an analogous expression for $\varphi^*$. The quantities $\langle \varphi\varphi \rangle$ and $\langle \varphi^*\varphi^* \rangle$ are usually known as the anomalous density terms, while $\langle \varphi^*\varphi \rangle$ is the so-called non-condensate density.

In the mean field (or self-consistent HF) approach used here, we restrict ourselves to the dynamics in the collisionless regime (see e.g. Ref. [10]), valid when the frequency of the excitations, $\omega_{\mathrm{exc}}$, is much larger than the typical collision (or relaxation) time, $t_{\mathrm{coll}}$: $\omega_{\mathrm{exc}}t_{\mathrm{coll}} \gg 1$. In the opposite regime, $\omega_{\mathrm{exc}}t_{\mathrm{coll}} \ll 1$, called hydrodynamical regime, the dynamics happens in the collision-dominated regime in which we can no longer neglect three-field contributions like $\langle \varphi^*\varphi\varphi^* \rangle$ that would enter in the equation of motion for the condensate and fluctuations. These terms describe collisional effects that become important for temperatures close to the critical

temperature of transition, and must be included in order to reproduce the results of the experiments of condensation dynamics in a trap. They can be introduced, for example, in the context of quantum Boltzmann equations, and lead to a consistent description of the evaporative cooling and subsequent formation and equilibration of the condensate, leading to a consistent description of the experimental data available, as shown in Refs. [5,6]. As we will discuss below, the parameters we use fall deeply into the collisionless regime, warranting the validity of the mean-field approach.

In Eq. (2), the chemical potential in the HF approximation is given by [11] $\mu_{\mathrm{HF}} = g(n + \langle \varphi^*\varphi \rangle + \langle \varphi\varphi \rangle)$. It however does not satisfy the Hugenholtz-Pines (HP) theorem for the chemical potential, $\mu_{\mathrm{HP}} = g(n + \langle \varphi^*\varphi \rangle - \langle \varphi\varphi \rangle)$, resulting then in the existence of a gap in the spectrum of elementary excitations at equilibrium [12], in opposition to the known results for Bose condensates. Even though we treat the nonequilibrium evolution of the condensate where the Hugenholtz-Pines theorem, which is a purely static relation valid in equilibrium, is not relevant [13], the use of the HF approximation for the description of the noncondensate fluctuations violates the conservation of particle number, which in equilibrium results in the violation of the HP theorem. This is a symptom of the fact that this approximation does not respect the constraints on the dynamics imposed by the spontaneous breaking of a $U(1)$ symmetry.

On the other hand, in the Hartree-Fock-Bogoliubov-Popov (HFBP) approximation, where the anomalous density is neglected in Eq. (2), the theory is gapless [12]. This is the approximation used in Ref. [7], where the chemical potential was shown to be $\mu = 2gn$. One way to go beyond the HFBP approximation and at the same time keep the theory gapless, as proposed in Ref. [14], is to treat the (contact) coupling constant $g$ in the HFBP approximation as an effective coupling constant $g_{\mathrm{eff}}$ that can be defined as

$$g_{\mathrm{eff}} = g(1 + \langle \varphi\varphi \rangle/|\varphi_0|^2) . \qquad (3)$$

As shown by the authors in Ref. [14], this is a consistent way to include the effects of pair correlations, represented by the anomalous density, and accounts for the low momentum limit corrections of the many body T-matrix keeping the theory gapless.

Eq. (2) in the improved HFBP approximation together with the constraint on the total density completely determines the time evolution of the condensate, the noncondensate density $\langle \varphi^*\varphi \rangle$, as well as the anomalous density $\langle \varphi\varphi \rangle$ in $g_{\mathrm{eff}}$. In order to solve these equations numerically, it is convenient to rewrite them in terms of their real field components, $\varphi = \varphi_1 + i\varphi_2$ and its complex conjugate. The two-point function $\langle \varphi^*\varphi \rangle$ and the anomalous densities can be expressed in a convenient way in terms of the Green's functions $\langle \varphi_i\varphi_j \rangle$ $(i, j = 1, 2)$ constructed from the homogeneous solutions to the operator

of quadratic fluctuations. Using the usual Schwinger-Keldysh formalism (see, *e.g.*, [15,13]) we find [7]

$$\langle\varphi_j(t)\varphi_j(t)\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{i}{1-\exp\left(-\beta\varepsilon_{\mathbf{k}}\right)} \left[\chi_j(\mathbf{k},t)\chi_j^*(\mathbf{k},t')\right.$$
$$\left. + \exp\left(-\beta\varepsilon_{\mathbf{k}}\right)\chi_j^*(\mathbf{k},t)\chi_j(\mathbf{k},t')\right] , \qquad (4)$$

where $\varepsilon_{\mathbf{k}} = \mathbf{k}^2/(2m)$, and $\beta^{-1}$ is the final temperature the system will reach at long times. The equations for the fluctuation modes, $\chi_1$ and $\chi_2$, that are the homogeneous solutions to the operator of quadratic fluctuations, were derived before [7] in the HFBP approximation. These mode equations, in the improved HFBP approximation discussed above, become

$$\frac{d\chi_2}{dt} + \left[\frac{\mathbf{k}^2}{2m} + g_{\text{eff}}|\varphi_0|^2\right]\chi_1 = 0 \quad ,$$
$$\frac{d\chi_1}{dt} - \left[\frac{\mathbf{k}^2}{2m} - g_{\text{eff}}|\varphi_0|^2\right]\chi_2 = 0 \quad , \qquad (5)$$

where $g_{\text{eff}}$ is given by Eq. (3). The boundary conditions for the solution of Eq. (5) are such that for $t < 0$, $|\varphi_0(t)|^2 = 0$ and $\chi_1(\mathbf{k},t)$ and $\chi_2(\mathbf{k},t)$ are given by $\chi_1(\mathbf{k},t) = \exp(i\varepsilon_{\mathbf{k}}t)/\sqrt{2N}$ and $\chi_2(\mathbf{k},t) = i\exp(i\varepsilon_{\mathbf{k}}t)/\sqrt{2N}$. The normalization factor $N$ is easily found to be [7] $N = 2(\beta_0/\beta)^{3/2}$ where $\beta_0^{-1}$ is the critical temperature for condensation of an ideal Bose gas.

The density constraint, when expressed as a solution for the condensate density $n_0(t) = |\varphi_0(t)|^2$, becomes [7]

$$|\varphi_0(t)|^2 = \frac{1}{2\pi^2}\left(\frac{\beta}{\beta_0}\right)^{3/2}\int_{k<k_c} dk\ k^2$$
$$\times \left[1 - N\left(|\chi_1(\mathbf{k},t)|^2 + |\chi_2(\mathbf{k},t)|^2\right)\right]n_{\mathbf{k}}(\beta) \quad , \quad (6)$$

where $n_{\mathbf{k}}(\beta) = (e^{\beta\varepsilon_{\mathbf{k}}} - 1)^{-1}$. The momentum integral in Eq. (6) is bound to values of $k < k_c$, corresponding to the unstable modes that drive the system from the initial excited state to the condensate state. $k_c$ can be determined from the equation for the modes, and is given by the value of $k$ which changes the sign in the product of the frequencies in Eq. (5).

In Fig. 1 we display the result for the evolution of the condensate density, as given by the solution for the coupled integro-differential system of equations (5) and (6), for three values of the final temperature. This result improves over the previous one, presented in Ref. [7], which was obtained in the HFBP approximation by neglecting the contribution from the anomalous densities. Although the HFBP approximation describes well the condensate for temperatures well below the critical temperature, it is well known that it breaks down for temperatures of the thermal bath near the critical one, in which case the anomalous densities become of the same magnitude as the out of the condensate excitation densities. In particular, we show in Fig. 2 that even for moderate temperatures the anomalous densities can grow to appreciable

values during the out-of-equilibrium evolution of the system.

From the results of Fig. 1 we have verified that, at large $t$, the temperature dependence of the condensate (in equilibrium) is the expected one, matching the known results for the drop of the condensate density in equilibrium for increasing temperatures in an interacting gas [2]. However, for initial times, this tendency changes due to pair correlation contributions represented by the anomalous density, $\langle\varphi\varphi\rangle$, in Eq. (5). This fact suggests the existence of two different scales for the equilibration time: one related to the condensate growth and the other to the anomalous density.

In order to investigate this behavior more precisely, let us consider the structure function defined as the Fourier transform of the two-point Green function, $S_k(t) = \langle\varphi_j(t)\varphi_j(t)\rangle_k$ [16] (this should not be confused with the usual dynamical structure function, given in terms of the Fourier transform of the four-point Green function instead). We find it more convenient to work with the normalized structure function, $S_k(t)/S_k(0)$, which, using Eq. (4), is given by $S_k(t)/S_k(0) = N\left(|\chi_1(\mathbf{k},t)|^2 + |\chi_2(\mathbf{k},t)|^2\right)$. The wave-number integrated time derivative of the normalized structure function provides the net change in the excitations outside the condensate, or the total change of fluctuations in the condensate. We define this quantity as

$$\Delta S_{\text{tot}}(t) = \left|\int_{k<k_c} d^3k \frac{\partial}{\partial t}\left[\frac{S_k(t)}{S_k(0)}\right]\right| \quad . \qquad (7)$$

As shown in Ref. [8], the peak of $\Delta S_{\text{tot}}(t)$ gives a measure of the departure from thermal equilibrium following a quench, while the position of the peak gives the equilibration time scale of the system. These properties can be extracted from Fig. 3, which shows $\Delta S_{\text{tot}}(t)$ for the three cases defined in Fig. 1.

In Fig. 3 we observe *two peaks* for each temperature studied. The first peak is clearly related to the time evolution of the anomalous density (compare with Fig. 2), and the second peak to the time evolution of the condensate density (compare with Fig. 1). Notice the correlation of the results regarding the time scale of equilibration for the condensate (associated to the position of the second peak). The lower the temperature of the thermal bath into which the system is quenched the higher is the peak of $\Delta S_{\text{tot}}(t)$. This is consistent with the fact that the lower the temperature of the thermal bath the further from equilibrium the system will be. The temperature dependence of the equilibration times is quite clear. For low temperatures, $T/T_0 < 0.1$, the first peak disappears. The equilibration process in this regime is completely driven by the condensation growth. In this case, the approximations used in Ref. [7] are acceptable since the anomalous density contribution becomes subleading. On the other hand, for final temperatures near

the critical one, $0.3 < T/T_0 < 1$, the second peak in Fig. 3 disappears and the equilibration process becomes now fully governed by thermal fluctuations encoded in the mean value of the anomalous density. Moreover, we find an intermediate temperature region, $0.1 < T/T_0 < 0.3$ (shown in Fig. 3), where the two equilibration times are present.

In Fig. 4 we exhibit the results for the equilibration time $t_{eq}$ ($\equiv (ma^2/\hbar)\tau_{eq}$) for both the condensate (squares) and the anomalous density (triangles) as a function of the (dimensionless) density $\rho = na^3$ at fixed temperature $T/T_0 = 0.1$, extracted from the location of the second and first peaks, respectively, of $\Delta S_{tot}$. The results can be perfectly fitted by the function $\tau_{eq,\varphi_0^2}^{fit} \simeq 0.2/\rho$ (line), for the condensate growth, while the result related to the equilibration of the anomalous density scales with the density in an analogous way, $\tau_{eq,\langle\varphi\varphi\rangle}^{fit} \simeq 0.12/\rho$ (dashed line). In terms of dimensionful quantities:

$$t_{eq} \simeq 2.5 \frac{\hbar}{ng} . \tag{8}$$

These results could in principle be compared with some early qualitative estimates for the growth time of the condensate for homogeneous gases [17], even though the physics discussed in those references is essentially different, yielding a rough estimate of the formation time $\sim O[(ng)^{-1}]$. This is exactly the behavior found here.

Although we deal with an idealized situation for the condensation formation, it is useful to briefly discuss the experimental results and compare them with our estimates, pointing out clearly the differences and similarities. Up to date only two experiments have been performed involving the explicit study of the dynamics of BEC formation. The first one was performed by the MIT group and reported in Ref. [3]. They have studied the dynamics of BEC of atomic $^{23}Na$ ($a = 2.75\,nm$) in a cigar shaped magnetic trap, described by frequencies $\omega_x = \omega_y = 2\pi\,82.3\,Hz$ and $\omega_z = 2\pi\,18\,Hz$. The number of atoms in the trap was $N \simeq 10^7$ and the temperatures involved were between $0.5 - 1.5\,\mu K$. The geometry of the confining potential yields a volume at the center of the trap [2] $V_{trap} = \pi^{3/2}a_x a_y a_z$, where $a_i = \sqrt{\hbar/(m\omega_i)}$. The central density can then be estimated as $n \sim 6.8 \times 10^{22}$ atoms/$m^3$, corresponding to $\rho = na^3 \sim 10^{-3}$. The typical formation time observed was $\sim 100 - 200\,ms$.

In the second experiment, Ref. [4], performed by the Munich group, they used $^{87}Rb$ ($a = 5.77\,nm$) in a trap characterized by $\omega_x = \omega_y = 2\pi\,110\,Hz$ and $\omega_z = 2\pi\,14\,Hz$. The number of atoms in the trap was $N \simeq 4.2 \times 10^6$ and the temperatures involved was $\sim 640\,nK$. These parameters lead to a central density value of $n \sim 2.5 \times 10^{23}$ atoms/$m^3$, corresponding to $\rho = na^3 \sim 5 \times 10^{-2}$. The typical formation time observed was also $\sim 200\,ms$.

From the experimental data above, we can estimate in which regime is each of the experiments. For trapped atom gas experiments $\omega_{exc}$ scales with the trapping potential frequency and $t_{col}$ is usually a measured quantity. Taking $\omega_{exc} \sim \bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$, the average frequency, and using the experimental data for the relaxation time measured in these experiments, we find for the MIT experiment that $\omega_{exc}t_{coll} \sim 3$, while for the Munich experiment $\omega_{exc}t_{coll} \sim 40$, putting these experiments in the borderline between the collisionless and hydrodynamical regimes (MIT) and in the collisionless regime (Munich). For a homogeneous gas it is more difficult to get such estimates. However, it is reasonable to estimate $\omega_{exc}$ as proportional to the thermal energy of the bath $\hbar\omega_{exc} \sim k_B T$ and $t_{coll}$ expressed in terms of the mean time between collisions, which for a classical gas is $(\sigma\bar{n}\bar{v})^{-1}$ (for an explicitly determination of the relaxation time, as given by the inverse of the damping rate for a homogeneous weakly interacting Bose gas, see Ref. [18]), where $\sigma = 8\pi a^2$ is the $s$-wave cross-section, $\bar{n}$ is the maximum non-condensate density and $\bar{v}$ is the characteristic thermal velocity. Using our parameter values and a temperature of $T = 0.1T_c$, with $T_c$ approximated by the condensation temperature for an ideal gas[1], we obtain $\omega_{exc}t_{coll} \sim 60$, deeply into the collisionless regime, for both $^{23}Na$ and $^{87}Rb$. The results for the equilibration times we obtain are of order $10^{-6}s$ for the parameters of $^{87}Rb$ and $10^{-7}s$ for $^{23}Na$.

The large difference in magnitude of time scales obtained here as compared to those of the experiments signals the importance of incoherent/coherent collisional effects in the trapped atomic gases experiments due to the nature of the experimental setup. Differently from our quenching scenario, where we can effectively approximate the model by a two-level system at low temperatures, in the experiments, the confined gas is firstly cooled to a temperature close to the critical temperature and then it is put in a nonequilibrium state by means of an evaporative cooling through a rapid rf sweep (MIT experiment) or a constant frequency rf field (Munich experiment), which removes the highly excited atoms from the trap. The system in then left to thermalize to a temperature below $T_c$ and the formation (growth) of the condensate occurs. This cannot be approximated by a two-level system. In the experiments, scattering and growth among adjacent levels, which are described by those microscopic processes associated with collisions, are of relevance to the dynamics in this regime. Those processes are indeed appropriately described through the use of quantum kinetic theory (quantum Boltzmann equations) for

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[1]For an estimate this is a reasonable approximation since for a dilute weakly interacting gas the the shift of the transition temperature, $\Delta T_c/T_c$, of the interacting model compared with the ideal gas transition temperature, is of order $an^{1/3}$ [19].

the trapped gases and are necessary to describe the whole process of evaporative cooling and subsequent thermalization during the process of condensate formation as shown in Refs. [5,6]. It is clear that we are considering a different regime from the beginning. In particular, we do not have a thermalization regime (since it is considered in thermal equilibrium from the beginning). Collisional effects lead to a much longer evolution for the condensate, explaining the disparities of orders of magnitude between our results and the experimental ones. We recall that, as pointed out by Stoof in Ref. [13], the regime we studied (named the second stage of condensate formation in that reference) lasts about $O(\hbar/ng)$, which corresponds exactly to the time scale we found, while the relaxation towards equilibrium in the third and final stage of the condensate formation (absent in our theoretical setting) leads to a much longer time evolution for the condensate.

In summary, we have provided in this paper a way to analyze the time scales related with the formation of the condensate, by studying the behavior of the total structure function, as defined by Eq. (7). We have analyzed the case of a homogeneous gas tha undergoes an instantaneous quench and we have shown that for temperatures of the thermal bath much lower than the critical temperature the gas is deep in the collisionless regime, justifying the mean field approach developed in this paper. We showed that under this circumstances, there are two equilibration time scales perfectly distinguished and associated with different physical processes, *i. e.* the growth of the condensate and the pair scattering process described by the anomalous density. The structure of equilibration times reported in this paper opens several questions related to the approach to equilibrium in interacting dilute gases. For instance, the presence of two equilibration time scales may influence time correlations, modifying fluctuation-dissipation relations. In order to address these issues, theoretically as well as experimentally, it is necessary to understand the behavior of correlation functions that depend on two different times as, for instance, the two-time dynamical structure factor. We hope to report on this subject in the near future. Another issue we are currently pursuing is the improvement of the equations presented here by including the higher-order collision terms by means of self-consistent Schwinger-Dyson equations for the nonequilibrium propagators. These results will be presented in the near future.

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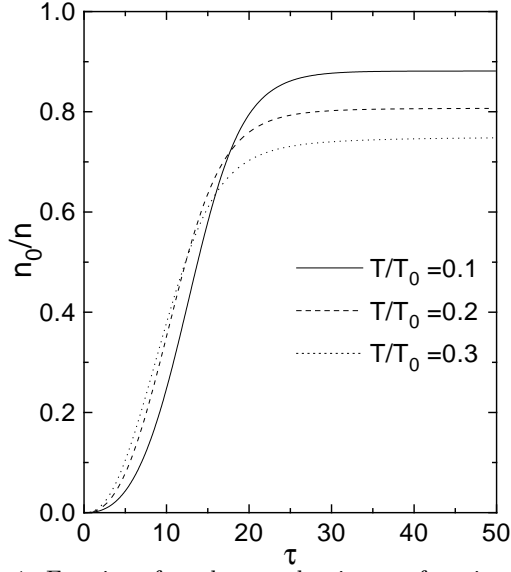
FIG. 1. Fraction of condensate density as a function of time for $na^3 = 0.01$ and $T/T_0 = 0.1, 0.2, 0.3$. Here, $\tau \equiv (\hbar/ma^2)t$ is a dimensionless time.
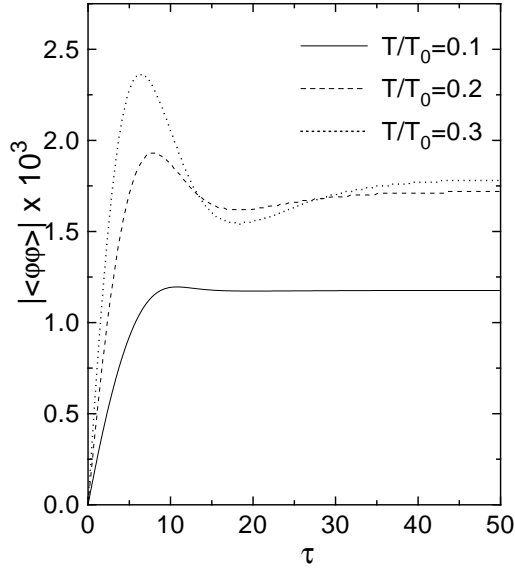


FIG. 2. Time evolution of the anomalous density for the same parameters used in Fig. 1.
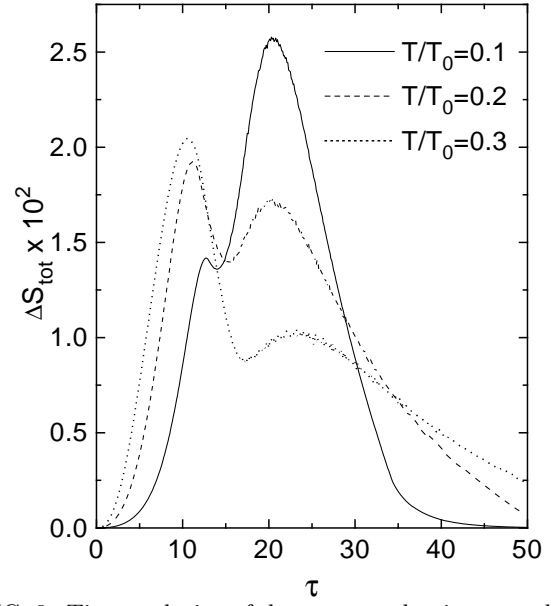


FIG. 3. Time evolution of the wave number integrated time derivative of the normalized structure function for the same parameters used in Fig. 1.
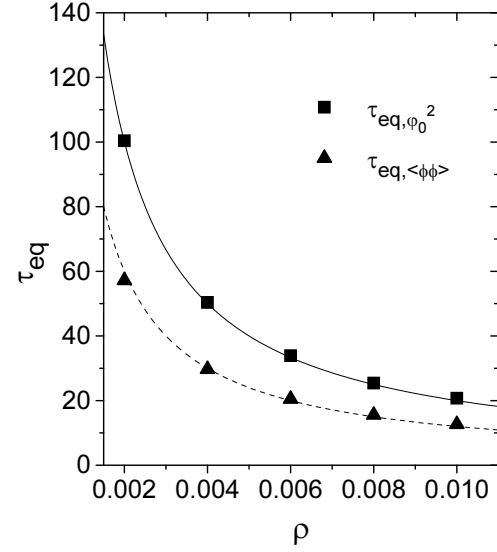


FIG. 4. Equilibration time for the condensate (squares) and anomalous density (triangles) obtained from the second and first peaks of $\Delta S_{\text{tot}}$, respectively, as a function of the density, $\rho = na^3$, at a fixed temperature $T/T_0 = 0.1$.